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More super Schrödinger algebras from psu(2,2|4)

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Abstract: We discuss super Schrödinger algebras with less supercharges from $\mathcal{N}$=4 superconformal algebra psu(2,2|4). Firstly $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras are constructed from the psu(2,2|4) via projection operators. Then a super Schrödinger subalgebra is found from each of them. The one obtained from $\mathcal{N}$=2 has 12 supercharges with su$(2)^2\times$u(1) and the other from $\mathcal{N}$=1 has 6 supercharges with u$(1)^3$. By construction, those are still subalgebras of psu(2,2|4). Another super Schrödinger algebra, which preserves 6 supercharges with a single u(1) symmetry, is also obtained from $\mathcal{N}$=1 superconformal algebra su(2,2|1). In particular, it coincides with the symmetry of $\mathcal{N}$=2 non-relativistic Chern-Simons matter system in three dimensions.

Keywords: Extended Supersymmetry, AdS-CFT Correspondence, Space-Time Symmetries.

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Contents

1. Introduction

A Schrödinger algebra [1, 2] is known as a non-relativistic analog of relativistic conformal algebra. The algebra with d spatial dimensions may be embedded into a conformal algebra so(d+2,2). Hence it is an interesting attempt to consider a role of the Schrödinger algebra in the context of AdS/CFT correspondence [3–5].

Non-relativistic CFT (NRCFT), which has Schrödinger symmetry, is discussed in [6–9] and it is expected to have an application to a cold atom system [10, 11]. A candidate of the gravity dual, which preserves the Schrödinger symmetry as the maximal one, is proposed in [10, 11]. As another scenario, it has been proposed in [12, 13] that one may consider AdS/NRCFT without deforming the metric and including any exotic matters. It would be an interesting direction to consider a supersymmetric extension of [12, 13] by considering the standard setup of AdS/CFT.

As the first step, super Schrödinger algebras should be found from the superconformal algebras. We have obtained the Schrödinger algebras with 24 supercharges as subalgebras of psu(2,2|4), osp(8|4) and osp(8*|4) [14], which preserve 16 rigid supersymmetries while half of 16 superconformal generators are projected out. The Schrödinger algebras may be realized in the corresponding gauge theories. But the field theoretical model, which has the super Schrödinger symmetry as the maximal one, has not been revealed yet.

The study of super Schrödinger algebra has a long history and it has been discussed in some contexts [15–19]. There are some models possessing super Schrödinger symmetry as the maximal one, such as a superparticle [15], super harmonic oscillator [16] and non-relativistic super Chern-Simons (CS) matter system in three dimensions [17].[1]

But the number of the conserved supercharges in the models is not so large and, for example, the non-relativistic CS matter system has $\mathcal{N}$=2 in three dimensions. On the other hand, our resulting algebras contain $\mathcal{N}$=8 in three dimensions. Hence, in order to find any common ground between our procedure and the existing result, it would be nice to look for less supersymmetric Schrödinger subalgebras of psu(2,2|4) . If we could find a point of agreement, it might give a clue to discuss the gauge-theory side.

In this manuscript we discuss this issue and find more super Schrödinger subalgebras of psu(2,2|4). We first construct $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras[2] from the psu(2,2|4) by constructing projection operators. Then a new super Schrödinger algebra is found from each of them. The one obtained from $\mathcal{N}$=2 contains 12 supercharges with su(2)$^2\times$u(1) R-symmetry and the other from $\mathcal{N}$=1 has 6 supercharges with u(1)^3 . Another super Schrödinger algebra, which preserves 6 supercharges with a single u(1) symmetry, is also obtained from $\mathcal{N}$=1 superconformal algebra su(2,2|1) . In particular, it coincides with the symmetry of the $\mathcal{N}$=2 non-relativistic CS matter system in three dimensions [17].

This manuscript is organized as follows. In section 2, $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras are derived from the $\mathcal{N}$=4 superconformal algebra psu(2,2|4) via projection operators. In section 3 super Schrödinger algebras are found as subalgebras of $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras. Section 4 is devoted to a conclusion and discussions. In the appendix we briefly summarize our notation and the relation between psu(2,2|4) and $\mathcal{N}$=4 superconformal generators.

2. $\mathcal{N}=2$ and 1 conformal algebras from psu(2,2|4)

By constructing projection operators, we will obtain $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras from the $\mathcal{N}$=4 superconformal algebra described by psu(2,2|4).[3]

2.1 $\mathcal{N}=4$ superconformal algebra

We begin with the four-dimensional $\mathcal{N}$=4 superconformal algebra. The commutation relations of the bosonic generators are composed of the AdS$_5$ part and the S^5 part. The AdS$_5$ part is given by[4]

$$
\begin{aligned}
&[\tilde{P}_\mu,\tilde{D}]=-\tilde{P}_\mu\,, &&[\tilde{K}_\mu,\tilde{D}]=\tilde{K}_\mu\,, &&[\tilde{P}_\mu,\tilde{K}_\nu]=\frac{1}{2}\tilde{J}_{\mu\nu}+\frac{1}{2}\eta_{\mu\nu}\tilde{D}\,,\\
&[\tilde{J}_{\mu\nu},\tilde{P}_\rho]=\eta_{\nu\rho}\tilde{P}_\mu-\eta_{\mu\rho}\tilde{P}_\nu\,, &&[\tilde{J}_{\mu\nu},\tilde{K}_\rho]=\eta_{\nu\rho}\tilde{K}_\mu-\eta_{\mu\rho}\tilde{K}_\nu\,, &&\qquad (2.1)\\
&[\tilde{J}_{\mu\nu},\tilde{J}_{\rho\sigma}]=\eta_{\nu\rho}\tilde{J}_{\mu\sigma}+\text{3-terms}\,,
\end{aligned}
$$

[1] The non-relativistic CS matter system was originally constructed by Jackiw and Pi [20], and its supersymmetrization has been done in [21].

[2] Those are obtained from $\mathcal{N}$=4 and hence should be called $\mathcal{N}$=2* and $\mathcal{N}$=1* , respectively. But for simplicity we will omit $*$ hereafter.

[3] For the relation between psu(2,2|4) and the generators of $\mathcal{N}$=4 superconformal, see appendix A.

[4] We suppress trivial (anti-)commutation relations below.

and the S^5 part is

$$[P_{a'}, P_{b'}] = -J_{a'b'}\,, \qquad [J_{a'b'}, P_{c'}] = \eta_{b'c'}P_{a'} - \eta_{a'c'}P_{b'}\,,$$
$$[J_{a'b'}, J_{c'd'}] = \eta_{b'c'}J_{a'd'} + \text{3-terms}\,.$$

Then the (anti-)commutation relations, which contain the fermionic generators $\tilde{Q}$ and $\tilde{S}$, are as follows. Those of the bosonic generators and the fermionic ones are

$$[\tilde{P}_\mu, \tilde{S}] = -\frac{1}{2}\tilde{Q}\Gamma_{\mu 4}\,, \qquad [\tilde{K}_\mu, \tilde{Q}] = \frac{1}{2}\tilde{S}\Gamma_{\mu 4}\,, \qquad [\tilde{D}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\,, \quad [\tilde{D}, \tilde{S}] = -\frac{1}{2}\tilde{S},$$
$$[\tilde{J}_{\mu\nu}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{\mu\nu}\,, \qquad [\tilde{J}_{\mu\nu}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{\mu\nu}\,,$$
$$[P_{a'}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{a'}\mathcal{J}i\sigma_2\,, \quad [J_{a'b'}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{a'b'}\,,$$
$$[P_{a'}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{a'}\mathcal{J}i\sigma_2\,, \quad [J_{a'b'}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{a'b'}\,,$$

and those of the fermionic generators are

$$\{\tilde{Q}^T, \tilde{Q}\} = 4iC\Gamma^\mu p_- h_+ \tilde{P}_\mu\,, \qquad \{\tilde{S}^T, \tilde{S}\} = 4iC\Gamma^\mu p_+ h_+ \tilde{K}_\mu\,,$$
$$\{\tilde{Q}^T, \tilde{S}\} = iC\Gamma^{\mu\nu}\mathcal{I}i\sigma_2 p_+ h_+ \tilde{J}_{\mu\nu} + 2iC\Gamma^4 p_+ h_+ \tilde{D}$$
$$+2iC\Gamma^{a'}p_+ h_+ P_{a'} - iC\Gamma^{a'b'}\mathcal{J}i\sigma_2 p_+ h_+ J_{a'b'}\,.$$

Here $\tilde{Q}$ are 16 supercharges while $\tilde{S}$ are 16 superconformal charges.

Next $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras will be obtained from the psu(2,2|4) by constructing projection operators.

2.2 $\mathcal{N} = 2$ superconformal algebra with su(2)$^2\times$u(1)

From now on we shall derive $\mathcal{N}$=2 superconformal algebra from $\mathcal{N}$=4 superconformal algebra.

For that purpose, let us introduce a projection operator defined by[5]

$$q_+ = \frac{1}{2}(1 + \Gamma^{5678})\,,$$

and require that

$$\tilde{Q} = \tilde{Q}q_+\,, \quad \tilde{S} = \tilde{S}q_+\,.$$

$\tilde{Q}$ are 8 supercharges while $\tilde{S}$ are 8 superconformal charges. Then the anti-commutation relation among $\tilde{Q}$ and $\tilde{S}$ are reduced to

$$\{\tilde{Q}^T, \tilde{Q}\} = 4iC\Gamma^\mu q_+ p_- h_+ \tilde{P}_\mu\,, \qquad \{\tilde{S}^T, \tilde{S}\} = 4iC\Gamma^\mu q_+ p_+ h_+ \tilde{K}_\mu\,,$$
$$\{\tilde{Q}^T, \tilde{S}\} = iC\Gamma^{\mu\nu}\mathcal{I}i\sigma_2 q_+ p_+ h_+ \tilde{J}_{\mu\nu} + 2iC\Gamma^4 q_+ p_+ h_+ \tilde{D}$$
$$+2iC\Gamma^9 q_+ p_+ h_+ P_9 - iC\Gamma^{\bar{a}'\bar{b}'}\mathcal{J}i\sigma_2 q_+ p_+ h_+ J_{\bar{a}'\bar{b}'}\,,$$

[5] Another projection operator $q_+ = \frac{1}{2}(1 + \Gamma^{56}i\sigma_2)$ also leads us to the similar result. In this case, su(2)$^2\times$u(1) R-symmetry is generated by $\{P_{\bar{a}'}, J_{\bar{a}'\bar{b}'}, J_{56}\}$ with $\bar{a}' = 7, 8, 9$.

where $\bar{a}' = 5, 6, 7, 8$.

One can find that the following set of the generators,

$$\{\tilde{K}_\mu, \tilde{S}, \tilde{D}, \tilde{J}_{\mu\nu}, P_9, J_{\bar{a}'\bar{b}'}, \tilde{Q}, \tilde{P}_\mu\} \tag{2.2}$$

forms $\mathcal{N} = 2$ superconformal algebra. Since $J_{\bar{a}'\bar{b}'}$ generates so(4)≅su(2)×su(2), the R-symmetry is su(2)×su(2)×u(1) generated by $\{P_9, J_{\bar{a}'\bar{b}'}\}$. The commutation relations between the bosonic generators are (2.1) and su(2)×su(2)×u(1), while those between the bosonic generators and $(\tilde{Q}, \tilde{S})$ are

$$\begin{aligned}
&[\tilde{P}_\mu, \tilde{S}] = -\frac{1}{2}\tilde{Q}\Gamma_{\mu 4}\,, \qquad [\tilde{K}_\mu, \tilde{Q}] = \frac{1}{2}\tilde{S}\Gamma_{\mu 4}\,, \qquad [\tilde{D}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\,, \quad [\tilde{D}, \tilde{S}] = -\frac{1}{2}\tilde{S}\,, \\
&[\tilde{J}_{\mu\nu}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{\mu\nu}\,, \qquad [\tilde{J}_{\mu\nu}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{\mu\nu}\,, \qquad (2.3) \\
&[P_9, \tilde{Q}] = \frac{1}{2}\tilde{Q}i\sigma_2\,, \qquad [J_{\bar{a}'\bar{b}'}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{\bar{a}'\bar{b}'}\,, \\
&[P_9, \tilde{S}] = \frac{1}{2}\tilde{S}i\sigma_2\,, \qquad [J_{\bar{a}'\bar{b}'}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{\bar{a}'\bar{b}'}\,. \qquad (2.4)
\end{aligned}$$

2.3 $\mathcal{N} = 1$ superconformal algebra with $\mathbf{u(1)}^3$

Here we derive $\mathcal{N}$=1 superconformal algebra from the psu(2,2|4).

Let us introduce a 1/4 projection operator defined by

$$q_+ = \frac{1}{2}(1 + \Gamma^{56}i\sigma_2)\frac{1}{2}(1 + \Gamma^{78}i\sigma_2)\,, \tag{2.5}$$

and require that

$$\tilde{Q} = \tilde{Q}q_+\,, \qquad \tilde{S} = \tilde{S}q_+\,.$$

Here $\tilde{Q}$ are 4 supercharges while $\tilde{S}$ are 4 superconformal charges. The anti-commutation relations of $\tilde{Q}$ and $\tilde{S}$ are reduced to

$$\begin{aligned}
\{\tilde{Q}^T, \tilde{Q}\} &= 4iC\Gamma^\mu q_+p_-h_+\tilde{P}_\mu\,, \qquad \{\tilde{S}^T, \tilde{S}\} = 4iC\Gamma^\mu q_+p_+h_+\tilde{K}_\mu\,, \\
\{\tilde{Q}^T, \tilde{S}\} &= iC\Gamma^{\mu\nu}\mathcal{I}i\sigma_2 q_+p_+h_+\tilde{J}_{\mu\nu} + 2iC\Gamma^4 q_+p_+h_+\tilde{D} - 2iC\mathcal{J}q_+p_+h_+R\,,
\end{aligned}$$

where we have relabeled the three generators as follows:

$$R = R_1 + R_2 + R_3\,, \quad R_I = (P_9, J_{56}, J_{78})\,.$$

Then one finds that the set of the generators,

$$\{\tilde{K}_\mu,\ \tilde{S},\ \tilde{D},\ \tilde{J}_{\mu\nu},\ R_I,\ \tilde{Q},\ \tilde{P}_\mu\}\,,$$

forms $\mathcal{N}$=1 superconformal algebra. Here R_I generate the R-symmetry $\mathrm{u}(1)^3$. The commutation relations of the bosonic generators are written in (2.1). Those of the bosonic generators and $(\tilde{Q}, \tilde{S})$ are given by

$$\begin{aligned}
&[\tilde{P}_\mu, \tilde{S}] = -\frac{1}{2}\tilde{Q}\Gamma_{\mu 4}\,, \qquad [\tilde{K}_\mu, \tilde{Q}] = \frac{1}{2}\tilde{S}\Gamma_{\mu 4}\,, \qquad [\tilde{D}, \tilde{Q}] = \frac{1}{2}\tilde{}, \quad [\tilde{D}, \tilde{S}] = -\frac{1}{2}\tilde{S}\,, \\
&[\tilde{J}_{\mu\nu}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{\mu\nu}\,, \qquad [\tilde{J}_{\mu\nu}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{\mu\nu}\,, \qquad (2.6) \\
&[R_I, \tilde{Q}] = -\frac{1}{2}\tilde{Q}i\sigma_2\,, \qquad [R_I, \tilde{S}] = -\frac{1}{2}\tilde{S}i\sigma_2\,. \qquad (2.7)
\end{aligned}$$

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Finally we note that

$$\{\tilde{K}_\mu,\ \tilde{S},\ \tilde{D},\ \tilde{J}_{\mu\nu},\ R,\ \tilde{Q},\ \tilde{P}_\mu\} \tag{2.8}$$

forms the superalgebra su(2,2|1).

3. Super Schrödinger algebras

It is turn to find super Schrödinger subalgebras of $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras constructed in the previous section.

First of all, we consider the bosonic part. The discussion is common in both $\mathcal{N}$=2 and $\mathcal{N}$=1 cases. In order to further reduce the commutation relations, let us decompose the bosonic generators as follows:

$$\begin{aligned}
&P_\pm = \frac{1}{\sqrt{2}}(\tilde{P}_0 \pm \tilde{P}_3)\,, \quad K_\pm = \frac{1}{\sqrt{2}}(\tilde{K}_0 \pm \tilde{K}_3)\,, \quad J_{i\pm} = \frac{1}{\sqrt{2}}(\tilde{J}_{i0} \pm \tilde{J}_{i3})\,, \\
&D = \frac{1}{2}(\tilde{D} - J_{03})\,, \qquad D' = \frac{1}{2}(\tilde{D} + J_{03})\,, \qquad P_i = \tilde{P}_i\,, \quad K_i = \tilde{K}_i\,, \quad J_{ij} = \tilde{J}_{ij} \\
&\mu = (0, i, 3), \qquad\qquad i = 1, 2.
\end{aligned} \tag{3.1}$$

Then it is straightforward to rewrite the commutation relations in (2.1) as

$$\begin{aligned}
&[J_{ij}, J_{k\pm}] = \eta_{jk} J_{i\pm} - \eta_{ik} J_{j\pm}\,, && [J_{i\pm}, J_{j\mp}] = J_{ij} \pm \eta_{ij}(D' - D)\,, \\
&[J_{ij}, P_k] = \eta_{jk} P_i - \eta_{ik} P_j\,, && [J_{ij}, K_k] = \eta_{jk} K_i - \eta_{ik} K_j\,, \\
&[P_i, K_j] = \frac{1}{2} J_{ij} + \frac{1}{2}\eta_{ij}(D' + D)\,, && [P_i, K_\pm] = \frac{1}{2} J_{i\pm}\,, \quad [P_\pm, K_i] = -\frac{1}{2} J_{i\pm}\,, \\
&[D, J_{i\pm}] = \mp\frac{1}{2} J_{i\pm}\,, && [D', J_{i\pm}] = \pm\frac{1}{2} J_{i\pm}\,, \\
&[P_i, J_{j\pm}] = \eta_{ij} P_\pm\,, && [K_i, J_{j\pm}] = \eta_{ij} K_\pm\,, \ [J_{i\pm}, P_\mp] = -P_i\,, \ [J_{i\pm}, K_\mp] = -K_i\,, \\
&[P_+, K_-] = -D'\,, && [P_-, K_+] = -D\,, \\
&[D, P_-] = P_-\,, && [D, P_i] = \frac{1}{2} P_i\,, \quad [D, K_+] = -K_+\,, \quad [D, K_i] = -\frac{1}{2} K_i\,, \\
&[D', P_+] = P_+\,, && [D', P_i] = \frac{1}{2} P_i\,, \quad [D', K_-] = -K_-\,, \quad [D', K_i] = -\frac{1}{2} K_i\ .
\end{aligned}$$

Here the set of the generators

$$\{J_{ij},\ J_{i+},\ D,\ P_\pm,\ P_i,\ K_+\} \tag{3.2}$$

is a subalgebra of so(4,2) and it forms the Schrödinger algebra

$$\begin{aligned}
&[J_{ij}, J_{k+}] = \eta_{jk} J_{i+} - \eta_{ik} J_{j+}\,, && [J_{ij}, P_k] = \eta_{jk} P_i - \eta_{ik} P_j\,, && [J_{i+}, P_-] = -P_i\,, \\
&[P_i, K_+] = \frac{1}{2} J_{i+}\,, && [P_i, J_{j+}] = \eta_{ij} P_+\,, && [P_-, K_+] = -D\,, \\
&[D, J_{i+}] = -\frac{1}{2} J_{i+}\,, && [D, P_-] = P_-\,, && [D, P_i] = \frac{1}{2} P_i\,, \quad [D, K_+] = -K_+\ .
\end{aligned} \tag{3.3}$$

We note that P_+ is a center of the Schrödinger algebra.

The remaining problem is the fermionic part, and hereafter we will discuss it by following the procedure developed in [14].

3.1 From $\mathcal{N}=2$ conformal algebra to super Schrödinger algebra

We consider the $\mathcal{N}$=2 superconformal algebra in this section.

According to the decomposition (3.1), we rewrite the (anti-)commutation relations including the fermionic generators as follows:

$$
\begin{aligned}
\{\tilde{Q}^T,\tilde{Q}\} &= 4iC\Gamma^+q_+p_-h_+P_+ + 4iC\Gamma^-q_+p_-h_+P_- + 4iC\Gamma^i q_+p_-h_+P_i\,,\\
\{\tilde{S}^T,\tilde{S}\} &= 4iC\Gamma^+q_+p_+h_+K_+ + 4iC\Gamma^-q_+p_+h_+K_- + 4iC\Gamma^i q_+p_+h_+K_i\,,\\
\{\tilde{Q}^T,\tilde{S}\} &= iC\Gamma^{ij}\mathcal{I}i\sigma_2 q_+p_+h_+J_{ij} + 2iC\Gamma^{i+}\mathcal{I}i\sigma_2 q_+p_+h_+J_{i+} + 2iC\Gamma^{i-}\mathcal{I}i\sigma_2 q_+p_+h_+J_{i-}\\
&\quad -2iC\Gamma^4\Gamma^+\Gamma^-q_+p_+h_+D' - 2iC\Gamma^4\Gamma^-\Gamma^+q_+p_+h_+D\\
&\quad +2iC\Gamma^9 q_+p_+h_+P_9 - iC\Gamma^{\bar{a}'\bar{b}'}\mathcal{J}i\sigma_2 q_+p_+h_+J_{\bar{a}'\bar{b}'}\,,
\end{aligned}
$$

$$
\begin{aligned}
&[K_\pm,\tilde{Q}] = -\frac{1}{2}\tilde{S}\Gamma^\mp\Gamma_4\,, \quad [K_i,\tilde{Q}] = \frac{1}{2}\tilde{S}\Gamma_{i4}\,, \quad [P_\pm,\tilde{S}] = \frac{1}{2}\tilde{Q}\Gamma^\mp\Gamma_4\,, \quad [P_i,\tilde{S}] = -\frac{1}{2}\tilde{Q}\Gamma_{i4}\,,\\
&[J_{ij},\tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{ij}\,, \quad [J_{ij},\tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{ij}\,, \quad [J_{i\pm},\tilde{Q}] = -\frac{1}{2}\tilde{Q}\Gamma_i\Gamma^\mp\,, \quad [J_{i\pm},\tilde{S}] = -\frac{1}{2}\tilde{S}\Gamma_i\Gamma^\mp\,,\\
&[D,\tilde{Q}] = -\frac{1}{4}\tilde{Q}\Gamma^+\Gamma^-\,, \quad [D,\tilde{S}] = \frac{1}{4}\tilde{S}\Gamma^-\Gamma^+\,, \quad [D',\tilde{Q}] = -\frac{1}{4}\tilde{Q}\Gamma^-\Gamma^+\,, \quad [D',\tilde{S}] = \frac{1}{4}\tilde{S}\Gamma^+\Gamma^-\,,
\end{aligned}
$$

and (2.4), where we have defined $\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^3)$.

As was seen above, the set of the generators (3.2) forms the Schrödinger algebra. Then we derive a super-Schrödinger algebra from $\mathcal{N}$=2 superconformal algebra below.

Let us introduce the light-cone projection operator

$$
\ell_\pm = \frac{1}{2}(1 \pm \Gamma^{03}) = -\frac{1}{2}\Gamma^\pm\Gamma^\mp\,, \tag{3.4}
$$

which commutes with projectors h_+, $p_\pm$ and $q_\pm$, and decompose $\tilde{S}$ as

$$
S = \tilde{S}\ell_-\,, \quad S' = \tilde{S}\ell_+\,. \tag{3.5}
$$

Then we show that

$$
\{J_{ij},\ J_{i+},\ D,\ P_\pm,\ P_i,\ K_+,\ \tilde{Q},\ S,\ P_9,\ J_{\bar{a}'\bar{b}'}\} \tag{3.6}
$$

forms a super Schrödinger algebra.

First the anti-commutation relations between $\tilde{Q}$ and S are given by

$$
\begin{aligned}
\{\tilde{Q}^T,\tilde{Q}\} &= 4iC\Gamma^+q_+p_-h_+P_+ + 4iC\Gamma^-q_+p_-h_+P_- + 4iC\Gamma^i q_+p_-h_+P_i\,,\\
\{S^T,S\} &= 4iC\Gamma^+\ell_-q_+p_+h_+K_+\,,\\
\{\tilde{Q}^T,S\} &= iC\Gamma^{ij}\mathcal{I}i\sigma_2\ell_-q_+p_+h_+J_{ij} + 2iC\Gamma^{i+}\mathcal{I}i\sigma_2\ell_-q_+p_+h_+J_{i+}\\
&\quad -2iC\Gamma^4\Gamma^-\Gamma^+\ell_-q_+p_+h_+D\\
&\quad +2iC\Gamma^9\ell_-q_+p_+h_+P_9 - iC\Gamma^{\bar{a}'\bar{b}'}\mathcal{J}i\sigma_2\ell_-q_+p_+h_+J_{\bar{a}'\bar{b}'}\,,
\end{aligned} \tag{3.7}
$$

where we have used $\Gamma^\pm\ell_\pm = 0$. In the right-hand sides, the bosonic generators contained in (3.6) appear.

Next we examine commutation relations between the bosonic generators in (3.6) and $(\tilde{Q}, S)$

$$\begin{aligned}
&[K_+,\tilde{Q}] = -\frac{1}{2}S\Gamma^-\Gamma_4\,, && [P_-,S] = \frac{1}{2}\tilde{Q}\ell_+\Gamma^+\Gamma_4\,, && [P_i,S] = -\frac{1}{2}\tilde{Q}\ell_-\Gamma_{i4}\,,\\
&[J_{ij},\tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{ij}\,, && [J_{ij},S] = \frac{1}{2}S\Gamma_{ij}\,, && [J_{i+},\tilde{Q}] = -\frac{1}{2}\tilde{Q}\Gamma_i\Gamma^-\,,\\
&[D,\tilde{Q}] = -\frac{1}{4}\tilde{Q}\Gamma^+\Gamma^-\,, && [D,S] = \frac{1}{4}S\Gamma^-\Gamma^+\,, && \\
&[P_9,\tilde{Q}] = \frac{1}{2}\tilde{Q}i\sigma_2\,, && [J_{\bar{a}'\bar{b}'},\tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{\bar{a}'\bar{b}'}\,, && \\
&[P_9,S] = \frac{1}{2}Si\sigma_2\,, && [J_{\bar{a}'\bar{b}'},S] = \frac{1}{2}S\Gamma_{\bar{a}'\bar{b}'}\,. &&
\end{aligned} \tag{3.8}$$

The right-hand sides of the commutation relations above contain $\tilde{Q}$ and S only. Thus we find that (3.6) forms a super Schrödinger algebra. The bosonic subalgebra is a direct sum of the Schrödinger algebra and su(2)$^2\times$u(1). The number of the supercharges is 12 since we have projected out 1/4 supercharges of 16 fermionic generators of $\mathcal{N}$=2 superconformal algebra.

We note that the set of generators, (3.2), su(2)$^2\times$u(1) generators and $Q = \tilde{Q}\ell_-$, forms a super Schrödinger algebra with 4 supercharges. It is still a superalgebra even if there are no su(2)$^2\times$u(1) generators. Such a superalgebra is a superextension of the Schrödinger algebra with 4 supercharges.

3.2 From $\mathcal{N} = 1$ conformal algebra to super Schrödinger algebra

We consider the $\mathcal{N}$=1 superconformal algebra here.

Under the decomposition (3.1), the commutation relations, which include the fermionic generators, are

$$\begin{aligned}
\{\tilde{Q}^T,\tilde{Q}\} &= 4iC\Gamma^+q_+p_-h_+P_+ + 4iC\Gamma^-q_+p_-h_+P_- + 4iC\Gamma^iq_+p_-h_+P_i\,,\\
\{\tilde{S}^T,\tilde{S}\} &= 4iC\Gamma^+q_+p_+h_+K_+ + 4iC\Gamma^-q_+p_+h_+K_- + 4iC\Gamma^iq_+p_+h_+K_i\,,\\
\{\tilde{Q}^T,\tilde{S}\} &= iC\Gamma^{ij}\mathcal{I}i\sigma_2q_+p_+h_+J_{ij} + 2iC\Gamma^{i+}\mathcal{I}i\sigma_2q_+p_+h_+J_{i+} + 2iC\Gamma^{i-}\mathcal{I}i\sigma_2q_+p_+h_+J_{i-}\\
&\quad -2iC\Gamma^4\Gamma^+\Gamma^-q_+p_+h_+D' - 2iC\Gamma^4\Gamma^-\Gamma^+q_+p_+h_+D\\
&\quad -2iC\mathcal{J}q_+p_+h_+R\,,\\
[K_\pm,\tilde{Q}] &= -\frac{1}{2}\tilde{S}\Gamma^\mp\Gamma_4\,, \qquad [K_i,\tilde{Q}] = \frac{1}{2}\tilde{S}\Gamma_{i4}\,,\\
[P_\pm,\tilde{S}] &= \frac{1}{2}\tilde{Q}\Gamma^\mp\Gamma_4\,, \qquad [P_i,\tilde{S}] = -\frac{1}{2}\tilde{Q}\Gamma_{i4}\,,\\
[J_{ij},\tilde{Q}] &= \frac{1}{2}\tilde{Q}\Gamma_{ij}\,, \qquad [J_{ij},\tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{ij}\,,\\
[J_{i\pm},\tilde{Q}] &= -\frac{1}{2}\tilde{Q}\Gamma_i\Gamma^\mp\,, \qquad [J_{i\pm},\tilde{S}] = -\frac{1}{2}\tilde{S}\Gamma_i\Gamma^\mp\,,\\
[D,\tilde{Q}] &= -\frac{1}{4}\tilde{Q}\Gamma^+\Gamma^-\,, \qquad [D,\tilde{S}] = \frac{1}{4}\tilde{S}\Gamma^-\Gamma^+\,,\\
[D',\tilde{Q}] &= -\frac{1}{4}\tilde{Q}\Gamma^-\Gamma^+\,, \qquad [D',\tilde{S}] = \frac{1}{4}\tilde{S}\Gamma^+\Gamma^-\,,
\end{aligned} \tag{3.9}$$

and (2.7), where we have defined $\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^3)$.

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As was seen above, (3.2) is the Schrödinger algebra. Then we derive a supersymmetric extension of the algebra from $\mathcal{N}=1$ superconformal algebra below.

Let us introduce the light-cone projection operator (3.4) and decompose $\tilde{S}$ as (3.5). Then we find that

$$\{J_{ij},\ J_{i+},\ D,\ P_{\pm},\ P_i,\ K_+,\ \tilde{Q},\ S,\ R_I\} \tag{3.10}$$

forms a super Schrödinger algebra.

First derive anti-commutation relations between $\tilde{Q}$ and S

$$\begin{aligned}
\{\tilde{Q}^T,\tilde{Q}\} &= 4iC\Gamma^+ q_+p_-h_+P_+ + 4iC\Gamma^- q_+p_-h_+P_- + 4iC\Gamma^i q_+p_-h_+P_i\,,\\
\{S^T,S\} &= 4iC\Gamma^+\ell_-q_+p_+h_+K_+\,,\\
\{\tilde{Q}^T,S\} &= iC\Gamma^{ij}\mathcal{I}i\sigma_2\ell_-q_+p_+h_+J_{ij} + 2iC\Gamma^{i+}\mathcal{I}i\sigma_2\ell_-q_+p_+h_+J_{i+}\\
&\quad -2iC\Gamma^4\Gamma^-\Gamma^+\ell_-q_+p_+h_+D - 2iC\mathcal{J}\ell_-q_+p_+h_+R\,,
\end{aligned}$$

where we have used $\Gamma^{\pm}\ell_{\pm}=0$. In the right-hand sides, the bosonic generators in (3.10) appear.

Next we examine the commutation relations between the bosonic generators in (3.10) and $(\tilde{Q},S)$

$$\begin{aligned}
&[K_+,\tilde{Q}] = -\frac{1}{2}S\Gamma^-\Gamma_4\,, && [P_-,S] = \frac{1}{2}\tilde{Q}\ell_+\Gamma^+\Gamma_4\,, && [P_i,S] = -\frac{1}{2}\tilde{Q}\ell_-\Gamma_{i4}\,,\\
&[J_{ij},\tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{ij}\,, && [J_{ij},S] = \frac{1}{2}S\Gamma_{ij}\,, && [J_{i+},\tilde{Q}] = -\frac{1}{2}\tilde{Q}\Gamma_i\Gamma^-\,,\\
&[D,\tilde{Q}] = -\frac{1}{4}\tilde{Q}\Gamma^+\Gamma^-\,, && [D,S] = \frac{1}{4}S\Gamma^-\Gamma^+\,, &&\\
&[R_I,\tilde{Q}] = -\frac{1}{2}\tilde{Q}i\sigma_2\,, && [R_I,S] = -\frac{1}{2}Si\sigma_2\,. &&
\end{aligned} \tag{3.11}$$

The right-hand sides of (3.11) contain $\tilde{Q}$ and S only. Thus the set of the generators (3.10) forms a super Schrödinger subalgebra. The bosonic subalgebra is a direct sum of the Schrödinger algebra and $\mathrm{u}(1)^3$. The number of the supercharges is 6 since we have projected out 1/4 supercharges of 8 fermionic generators of $\mathcal{N}$=1 superconformal algebra.

We note that the set of generators, (3.2), R_I and $Q=\tilde{Q}\ell_-$, forms a super Schrödinger algebra with 2 supercharges. It is still a superalgebra even if there are no R_I. Such a superalgebra is a superextension of the Schrödinger algebra with 2 supercharges.

If we start from $\mathcal{N}$=1 superconformal algebra su(2,2|1) in (2.8), we obtain a super Schrödinger algebra with u(1) R-symmetry

$$\{J_{ij},\ J_{i+},\ D,\ P_{\pm},\ P_i,\ K_+,\ \tilde{Q},\ S,\ R\}\ . \tag{3.12}$$

A relation to non-relativistic CS matter system

It is worth noting the relation between the algebra (3.12) and $\mathcal{N}=2$ super Schrödinger algebra constructed in [17].

The bosonic subalgebra of (3.12) coincides with the bosonic part of the superalgebra in [17] under the following identification of the generators,

$$(J_{ij},P_-,P_i,J_{i+},P_+,D,K_+,R) = \left(J,H,P_i,G_i,N_B+N_F,D,K,N_B-\frac{1}{2}N_F\right)\,, \tag{3.13}$$

up to trivial scalings of generators. Here the right-hand side represents the generators used in [17].

Then the next task is to consider the fermionic part of the algebra. Our supercharges are Majorana-Weyl spinors in (9+1)-dimensions satisfying the Majorana condition $\mathcal{Q}^c = \mathcal{Q}$, as explained in appendix.

Note that we may choose the charge conjugation matrix as $C = \Gamma^0$ and then $B = 1$. It implies that Γ_A's are real: $\Gamma_A^* = \Gamma_A$. With this choice, the Majorana condition simply implies that $\mathcal{Q}^* = \mathcal{Q}$. Since the projectors $p_\pm$, q_+ and $\ell_\pm$ are real, the two-component spinors, Q, Q' and S are real: $Q^* = Q$, $Q'^* = Q'$ and $S^* = S$ where $Q = \tilde{Q}\ell_-$ and $Q' = \tilde{Q}\ell_+$.

The supercharges used in [17] are complex and hence it is necessary to convert our two-component real supercharges into one-component complex supercharges. For this purpose, let us introduce a pair of projectors

$$k_\pm = \frac{1}{2}(1 \pm i\Gamma^{12})$$

and decompose the two-component real spinors as

$$q_1 = Qk_+\,, \quad q_1' = Qk_-\,, \quad q_2 = Q'k_-\,, \quad q_2' = Q'k_+\,, \quad q_3 = Sk_-\,, \quad q_3' = Sk_+\,.$$

Noting that $k_\pm$ are complex: $k_\pm^* = k_\mp$, the Majorana condition implies that q_1, q_2 and q_3 are complex one-component spinors

$$q_1^* = q_1'\,, \qquad q_2^* = q_2'\,, \qquad q_3^* = q_3'\,.$$

With the complex supercharges, the anti-commutation relations are rewritten as

$$\begin{aligned}
\{q_1, q_1^*\} &= 4iC\Gamma^+k_-\ell_-q_+p_-h_+P_+\,,\\
\{q_2, q_2^*\} &= 4iC\Gamma^-k_+\ell_+q_+p_-h_+P_-\,,\\
\{q_1, q_2^*\} &= 4iC\Gamma^1k_+\ell_+q_+p_-h_+(P_1 - iP_2)\,,\\
\{q_3, q_3^*\} &= 4iC\Gamma^+k_-\ell_-q_+p_+h_+K_+\,,\\
\{q_1^*, q_3\} &= 2iC\Gamma^1\mathcal{I}i\sigma_2k_-\ell_-q_+p_+h_+(J_{1+} + iJ_{2+})\,,\\
\{q_2^*, q_3\} &= iC\Gamma^{ij}\mathcal{I}i\sigma_2k_-\ell_-q_+p_+h_+J_{ij} - 2iC\Gamma^4\Gamma^-\Gamma^+k_-\ell_-q_+p_+h_+D\\
&\quad -2iC\Gamma^4\Gamma^-\Gamma^+k_-\ell_-q_+p_+h_+R,
\end{aligned}$$

where we have used $k_\pm = \pm i\Gamma^{12}k_\pm$. On the other hand, the commutation relations including the supercharges are

$$\begin{aligned}
&[J_{12}, q_1] = -\frac{i}{2}q_1\,, && [J_{12}, q_2] = \frac{i}{2}q_2\,, && [J_{12}, q_3] = \frac{i}{2}q_3\,,\\
&[D, q_2] = -\frac{1}{4}q_2\Gamma^+\Gamma^-\,, && [D, q_3] = \frac{1}{4}q_3\Gamma^-\Gamma^+\,, && [J_{1+} - iJ_{2+}, q_2] = -q_1\Gamma_1\Gamma^-\,,\\
&[P_-, q_3] = \frac{1}{2}q_2\Gamma^+\Gamma_4\,, && [P_1 - iP_2, q_3] = -q_1\Gamma_{14}\,, && [K_+, q_2] = -\frac{1}{2}q_3\Gamma^-\Gamma_4\,,\\
&[R, q_1] = -\frac{1}{2}q_1i\sigma_2\,, && [R, q_2] = -\frac{1}{2}q_2i\sigma_2\,, && [R, q_3] = -\frac{1}{2}q_3i\sigma_2\,.
\end{aligned}$$

Under the identification (3.13), the last three commutation relations are further rewritten, by noting that P_+ is center of the super Schrödinger algebra, as

$$[N_F, q_I] = \frac{1}{3} q_I i\sigma_2 \,, \qquad [N_B, q_I] = -\frac{1}{3} q_I i\sigma_2 \qquad (I = 1, 2, 3)\,.$$

Thus we have shown that the above (anti-)commutation relations coincide with those of [17] under the identification (3.13) and $(q_1, q_2, q_3) = (Q_1, Q_2, F)$, up to trivial rescalings of generators.

4. Conclusion and discussion

We have found more super Schrödinger subalgebras of psu(2,2|4). First $\mathcal{N}$=2 and $\mathcal{N}$=1 superconformal algebras have been constructed from the psu(2,2|4) by constructing projection operators. Then a less supersymmetric Schrödinger algebra has been found from each of them. The resulting two superalgebras are as follows: the one preserves 12 supercharges with su$(2)^2\times$u(1) symmetry, and the other preserves 6 supercharges with u$(1)^3$ symmetry.

We have also found another super Schrödinger algebra preserving 6 supercharges with a single u(1) symmetry from su(2,2|1). This algebra coincides with the symmetry of the non-relativistic CS matter system in three dimensions [17].

It is interesting to look for non-relativistic systems which preserve super Schrödinger symmetry with 24 and 12 supercharges as the maximal ones (i.e., new super NRCFTs). Perhaps there would be two possible scenarios. The one is to reduce a four-dimensional superconformal field theory with a light-like compactification to a three-dimensional theory, according to the embedding of the Schrödinger algebra into the superconformal one. The other is to take the standard non-relativistic limit of certain relativistic models as in [17]. It would also be interesting to consider the gravity dual of the non-relativistic CS matter system.

We hope that our results would be a key to open a new arena to study the AdS/NRCFT correspondence.

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A. psu(2,2|4) as $\mathcal{N} = 4$ superconformal algebra

We briefly explain the relation between psu(2,2|4) and the generators of the $\mathcal{N}$=4 superconformal algebra. The commutation relations of the psu(2,2|4) are as follows. The bosonic part is composed of the so(2,4) algebra

$$[P_a, P_b] = J_{ab}\,, \qquad [J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b\,,$$
$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} + \text{3-terms} \qquad (a = 0,1,2,3,4)\,, \tag{A.1}$$

and the so(6) algebra

$$[P_{a'}, P_{b'}] = -J_{a'b'}\,, \qquad [J_{a'b'}, P_{c'}] = \eta_{b'c'} P_{a'} - \eta_{a'c'} P_{b'}\,,$$
$$[J_{a'b'}, J_{c'd'}] = \eta_{b'c'} J_{a'd'} + \text{3-terms} \qquad (a' = 5,6,7,8,9)\,. \tag{A.2}$$

The (anti-)commutation relations, which include the fermionic generator Q, are

$$\begin{aligned}
[P_a, \mathcal{Q}] &= -\frac{1}{2}\mathcal{Q}\Gamma_a \mathcal{I} i\sigma_2\,, \quad [P_{a'}, \mathcal{Q}] = \frac{1}{2}\mathcal{Q}\Gamma_{a'} i\sigma_2\,, \quad [J_{AB}, \mathcal{Q}] = \frac{1}{2}\mathcal{Q}\Gamma_{AB}\,,\\
\{\mathcal{Q}^T, \mathcal{Q}\} &= 2iC\Gamma^A h_+ P_A + iC\Gamma^{ab}\mathcal{I} i\sigma_2 h_+ J_{ab} - iC\Gamma^{a'b'}\mathcal{J} i\sigma_2 h_+ J_{a'b'}\,, \qquad A = (a, a')\\
\mathcal{I} &= \Gamma^{01234}\,, \qquad \mathcal{J} = \Gamma^{56789}\,.
\end{aligned}$$

Here Γ^A's are $(9+1)$-dimensional gamma-matrices and C is the charge conjugation matrix satisfying

$$\Gamma_A^T = -C\Gamma_A C^{-1}\,, \quad C^\dagger C = 1\,, \quad C^T = -C\,. \tag{A.3}$$

The supercharge $\mathcal{Q}$ is a pair of Majorana-Weyl spinors in $(9+1)$-dimensions. The charge conjugation of $\mathcal{Q}$ is defined by

$$\mathcal{Q}^c = \mathcal{Q}^* B^{-1}\,, \tag{A.4}$$

where the matrix B relates Γ_A^* and Γ_A by

$$\Gamma_A^* = B\Gamma_A B^{-1}\,, \quad B^\dagger B = 1\,, \quad B^T = B\,. \tag{A.5}$$

It is also related to C via

$$C = B\Gamma_0^\dagger\,.$$

The Majorana-Weyl spinor $\mathcal{Q}$ satisfies the Majorana condition

$$\mathcal{Q}^c = \mathcal{Q}$$

as well as the Weyl condition

$$\mathcal{Q} = \mathcal{Q}h_+\,, \qquad h_+ \equiv \frac{1}{2}(1 + \Gamma_{01\cdots 9}) \;:\; \text{chirality projector}\,.$$

By recombining the generators, we define the followings

$$\begin{aligned}
&\tilde{P}_\mu \equiv \frac{1}{2}(P_\mu - J_{\mu 4})\,, \qquad \tilde{K}_\mu \equiv \frac{1}{2}(P_\mu + J_{\mu 4})\,, \qquad \tilde{D} \equiv P_4\,, \quad \tilde{J}_{\mu\nu} \equiv J_{\mu\nu}\,,\\
&\tilde{Q} \equiv \mathcal{Q}p_-\,, \qquad \tilde{S} \equiv \mathcal{Q}p_+ \qquad (a = (\mu, 4), \quad \mu = 0,1,2,3)\,.
\end{aligned}$$

Here the projectors $p_\pm$ are defined by

$$p_\pm \equiv \frac{1}{2}(1 \pm \Gamma^4 \mathcal{I} i\sigma_2) = \frac{1}{2}(1 \pm \Gamma^{0123} i\sigma_2)\,, \tag{A.6}$$

which commute with the chirality projector h_+. By noting that

$$p_\pm^T C = C p_\pm\,, \qquad \Gamma^{0123} i\sigma_2 p_\pm = \pm p_\pm\,,$$

the (anti-)commutation relations can also be rewritten as

$$\begin{aligned}
[\tilde{P}_\mu, \tilde{D}] &= -\tilde{P}_\mu\,, \qquad [\tilde{K}_\mu, \tilde{D}] = \tilde{K}_\mu\,, \quad [\tilde{P}_\mu, \tilde{K}_\nu] = \frac{1}{2}\tilde{J}_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\tilde{D}\,,\\
[\tilde{J}_{\mu\nu}, \tilde{P}_\rho] &= \eta_{\nu\rho}\tilde{P}_\mu - \eta_{\mu\rho}\tilde{P}_\nu\,, \qquad [\tilde{J}_{\mu\nu}, \tilde{K}_\rho] = \eta_{\nu\rho}\tilde{K}_\mu - \eta_{\mu\rho}\tilde{K}_\nu\,,\\
[\tilde{J}_{\mu\nu}, \tilde{J}_{\rho\sigma}] &= \eta_{\nu\rho}\tilde{J}_{\mu\sigma} + \text{3-terms}\,,\\
\{\tilde{Q}^T, \tilde{Q}\} &= 4iC\Gamma^\mu p_- h_+ \tilde{P}_\mu\,, \qquad \{\tilde{S}^T, \tilde{S}\} = 4iC\Gamma^\mu p_+ h_+ \tilde{K}_\mu\,,\\
\{\tilde{Q}^T, \tilde{S}\} &= iC\Gamma^{\mu\nu}\mathcal{I} i\sigma_2 p_+ h_+ \tilde{J}_{\mu\nu} + 2iC\Gamma^4 p_+ h_+ \tilde{D}\\
&\quad + 2iC\Gamma^{a'} p_+ h_+ P_{a'} - iC\Gamma^{a'b'}\mathcal{J} i\sigma_2 p_+ h_+ J_{a'b'}\,,\\
[\tilde{P}_\mu, \tilde{S}] &= -\frac{1}{2}\tilde{Q}\Gamma_{\mu 4}\,, \qquad [\tilde{K}_\mu, \tilde{Q}] = \frac{1}{2}\tilde{S}\Gamma_{\mu 4}\,, \quad [\tilde{D}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\,, \quad [\tilde{D}, \tilde{S}] = -\frac{1}{2}\tilde{S}\\
[\tilde{J}_{\mu\nu}, \tilde{Q}] &= \frac{1}{2}\tilde{Q}\Gamma_{\mu\nu}\,, \qquad [\tilde{J}_{\mu\nu}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{\mu\nu}\,,\\
[P_{a'}, \tilde{Q}] &= \frac{1}{2}\tilde{Q}\Gamma_{a'}\mathcal{J} i\sigma_2\,, \qquad [J_{a'b'}, \tilde{Q}] = \frac{1}{2}\tilde{Q}\Gamma_{a'b'}\,,\\
[P_{a'}, \tilde{S}] &= \frac{1}{2}\tilde{S}\Gamma_{a'}\mathcal{J} i\sigma_2\,, \qquad [J_{a'b'}, \tilde{S}] = \frac{1}{2}\tilde{S}\Gamma_{a'b'}\,.
\end{aligned}$$

The so(6) part is given in (A.2). Thus the resulting algebra is nothing but the four-dimensional $\mathcal{N}$=4 superconformal algebra. Here $\tilde{Q}$ are 16 supercharges while $\tilde{S}$ are 16 superconformal charges.

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